

The Problem of "Material Implication"

The following issue is quite abstract and difficult, though I shall try to make it as simple as possible. It is included because I believe it shows that "something is rotten in the state of Denmark" at the very heart of the new logic. (For a fuller treatment of the new logic see the Appendix, p. 364.)

Logic is most especially about reasoning, or inference: the process of thinking by which we draw conclusions from evidence, moving from one proposition to another. The proposition we begin with is called a "premise" and the proposition we move to, or infer, or reason to, is called a "conclusion."

The simplest and most straightforward kind of reasoning is to move from a true premise (or, more usually, from a number of true premises together) to a true conclusion. But we can also use *false* propositions in good reasoning. Since a false conclusion cannot be logically proved from true premises, we can know that if the conclusion is false then one of the premises must also be false, in a logically valid argument.

A logically valid argument is one in which the conclusion necessarily follows from its premises. In a logically valid argument, if the premises are true then the conclusion must be true. In an invalid argument this is not so. "All men are mortal, and Socrates is a man, therefore Socrates is mortal" is a valid argument. "Dogs have four legs, and Lassie has four legs, therefore Lassie is a dog" is not a valid argument. The conclusion ("Lassie is a dog") may be true, but it has not been proved by this argument. It does not "follow" from the premises.

Now in Aristotelian logic, a true conclusion logically follows from, or is proved by, or is "implied" by, or is validly inferred from, only *some* premises and not others. The above argument about Lassie is *not* a valid argument according to Aristotelian logic. Its premises do not prove its conclusion. And common sense, or our innate logical sense, agrees. However, modern symbolic logic disagrees. One of its principles is that "if a statement is true, then that statement is implied by any statement whatever." Since it is true that Lassie is a dog, "dogs have four legs" implies that Lassie is a dog. In fact, "dogs do *not* have four legs" also implies that Lassie is a dog! Even false statements, even statements that are self-contradictory, like "Grass is not grass," validly imply any true conclusion in symbolic logic. And a second strange principle is that "if a statement is false, then it implies any statement whatever." "Dogs do not have four legs" implies that Lassie is a dog, and also that Lassie is not a dog, and that 2 plus 2 are 4, and that 2 plus 2 are not 4.

This principle is often called "the paradox of material implication." Ironically, "material implication" means exactly the opposite of what it seems to mean. It means that the matter, or content, of a statement is totally irrelevant to its logically implying or being implied by other statements. Common sense says that Lassie being a dog or not being a dog has nothing to do with 2+2 being 4 or not being 4, but that Lassie being a collie and collies being dogs does have something to do with Lassie being a dog. But not in the new logic, which departs from common sense here by totally sundering the rules for logical implication from the matter, or content, of the propositions involved. Thus, the paradox ought to be called "the paradox of *non*-material implication."

The paradox can be seen in the following imaginary conversation:

Logician: So, class, you see, if you begin with a false premise, anything follows.

Student: I just can't understand that.

Logician: Are you sure you don't understand that?

Student: If I understand that, I'm a monkey's uncle.

Logician: My point exactly. (Snickers.)

Student: What's so funny?

Logician: You just can't understand that.

The relationship between a premise and a conclusion is called "implication," and the process of reasoning from the premise to the conclusion is called "inference." In symbolic logic, the relation of implication is called "a truth-functional connective," which means that the only factor that makes the inference valid or invalid, the only thing that makes it true or false to say that the premise or premises validly imply the conclusion, is not at all dependent on the content or matter of any of those propositions, but only whether the premise or premises are true or false and whether the conclusion is true or false.

That last paragraph was cruelly abstract. Let's try to be a little more specific. In symbolic logic,

- (1) If the premise or premises (let's just say "the premise" for short) are true and the conclusion is true, then the "if . . . then" proposition summarizing the implication is true. If p is true and q is true, then "if p then q" is true. So "if grass is green, then Mars is red" is true.
- (2) If the premise is true and the conclusion is false, then the "if . . . then" proposition summarizing the implication is false. If p is true and q is false, then "if p then q" is false. So "if grass is green, then Mars is not red" is false.
- (3) If the premise is false and the conclusion is true, then the "if . . . then" proposition summarizing the implication is true. If p is false and q is true, then "if p then q" is true. So "if grass is purple, then Mars is red" is true.
- (4) If the premise is false and the conclusion is false, then the "if . . . then" proposition summarizing the implication is true. If p is false and q is false, then "if p then q" is true. So "if grass is purple, then Mars is purple" is also true!

In this logic, if the premise and the conclusion are both false, the premise implies the conclusion (this is #4), and if the premise is false and the conclusion is true, the premise also implies the conclusion (this is #3). So if the moon is blue, then the moon is red (#4); and if the moon is blue, then the moon is not blue (#3)! This may make some defensible sense mathematically, but it certainly does not make sense commonsensically, for it does not seem to make sense in the real world.

Logicians have an answer to the above charge, and the answer is perfectly tight and logically consistent. That is part of the problem! Consistency is not enough. Logic should be not just a mathematically consistent system but a human instrument for understanding reality, for dealing with real people and things and real arguments about the real world. That is the basic assumption of the old logic. If that assumption is naïve and uncritical, unfashionable and unintelligent — well, welcome to Logic for Dummies.